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CS1660: Intro to Computer Systems Security
Spring 2025

Lecture 6: Cryptography V

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CS1660: Announcements

- Course updates
  - Homework 1, Project 1 have new submission dates
  - Future assignment dates may be updated as well/accordingly
  - Ed Discussion, Top Hat (code: 821033), Gradescope (set up for Project 1)

We are fixing some issues with Autograder

Today

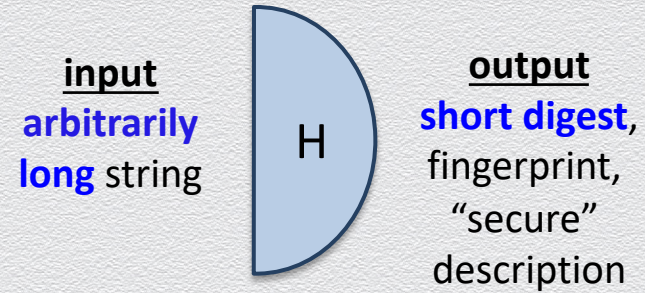
- Cryptography
  - Hash functions
    - Definition
    - Constructions
    - Generic attacks
    - Applications to cryptography
    - Applications to security

6.1 Cryptographic Hash functions

Cryptographic hash functions

Basic cryptographic primitive

- maps **objects** to a **fixed-length binary strings**
- core security property: mapping **avoids collisions**
  - **collision**: distinct objects ($x \neq y$) are mapped to the same hash value ($H(x) = H(y)$)
  - although collisions **necessarily exist**, they are **infeasible to find**



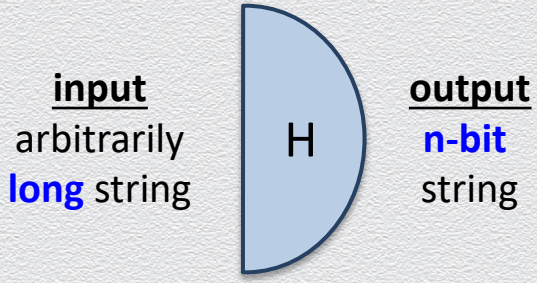
Important role in modern cryptography

- lie between symmetric- and asymmetric-key cryptography
- capture different security properties of "idealized random functions"
- qualitative stronger assumption than PRF

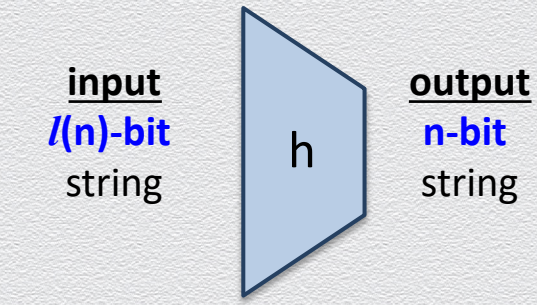
Hash & compression functions

Map messages to short digests

- a **general** hash function H() maps
  - a message of an **arbitrary length** to a **n-bit** string

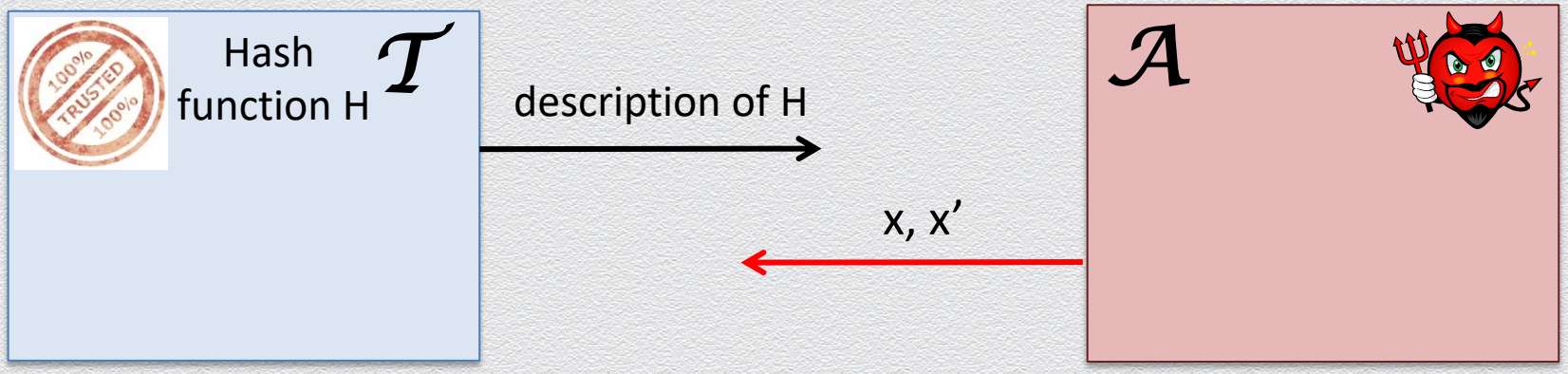


- a **compression** (hash) function h() maps
  - a **long** binary string to a **shorter** binary string
  - an ***l*(n)-bit string** to a **n-bit string**, with ***l*(n) > n**



Collision resistance (CR)

Attacker wins the game if $x \neq x'$ & $H(x) = H(x')$



H is collision-resistant if any PPT $\mathcal{A}$ wins the game only negligibly often.

Weaker security notions

Given a hash function H: X $\rightarrow$ Y, then we say that H is

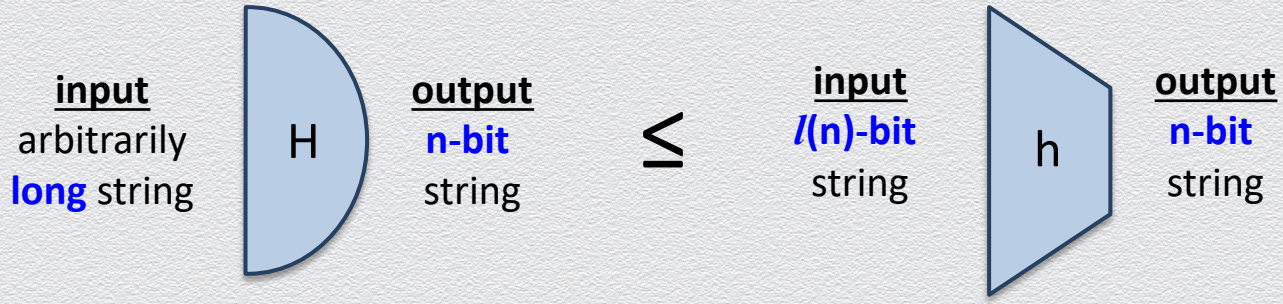
- **preimage resistant** (or **one-way**)
  - if given y $\in$ Y, finding a value x $\in$ X s.t. H(x) = y happens negligibly often
- **2-nd preimage resistant** (or **weak collision resistant**)
  - if given a **uniform** x $\in$ X, finding a value x' $\in$ X, s.t. x'$\neq$ x and H(x') = H(x) happens negligibly often
- **collision resistant** (or **strong collision resistant**)
  - if finding two distinct values x', x $\in$ X, s.t. H(x') = H(x) happens negligibly often

6.2 Design framework

Domain extension via the Merkle-Damgård transform

General design pattern for cryptographic hash functions

- reduces CR of general hash functions to CR of compression functions



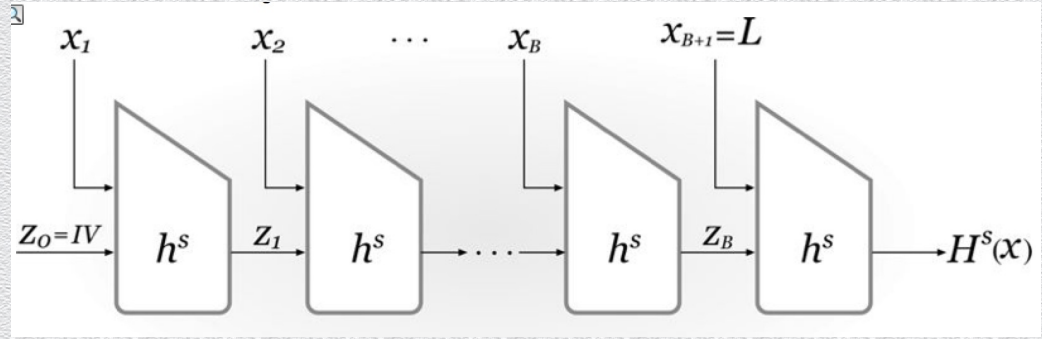
- thus, in practice, it suffices to realize a collision-resistant compression function h
- compressing by 1 single bit is a least as hard as compressing by any number of bits!

Merkle-Damgård transform: Design

Suppose that h: $\{0,1\}^{2n} \rightarrow \{0,1\}^n$ is a collision-resistant compression function

Consider the general hash function H: $\mathcal{M} = \{x : |x| < 2^n\} \rightarrow \{0,1\}^n$, defined as

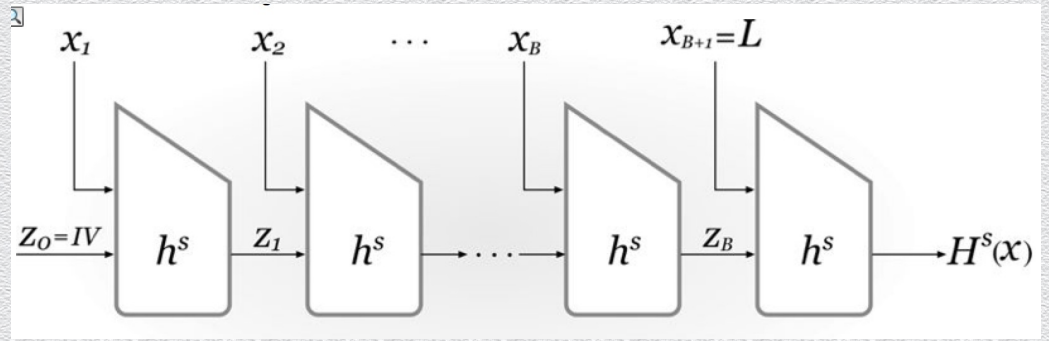
Merkle-Damgård design



- H(x) is computed by applying h() in a **"chained" manner** over n-bit message blocks
  - pad x to define a number, say B, **message blocks $x_1, \ldots, x_B$**, with $|x_i| = n$
  - set extra, final, message block **x_{B+1} as an n-bit encoding L of $|x|$**
  - starting by initial digest **$z_0 = IV = 0^n$**, output **$H(x) = z_{B+1}$**, where **$z_i = h^s(z_{i-1} || x_i)$**

Merkle-Damgård transform: Security

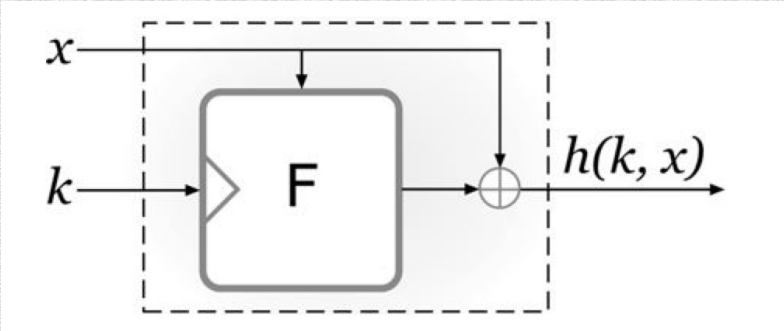
If the compression function h is CR,
then the derived hash function H is also CR!



Compression function design: The Davies-Meyer scheme

Employs PRF w/ key length m & block length n

- define h: $\{0,1\}^{n+m} \rightarrow \{0,1\}^{n}$ as **$h(x||k) = F_k(x)$ XOR x**



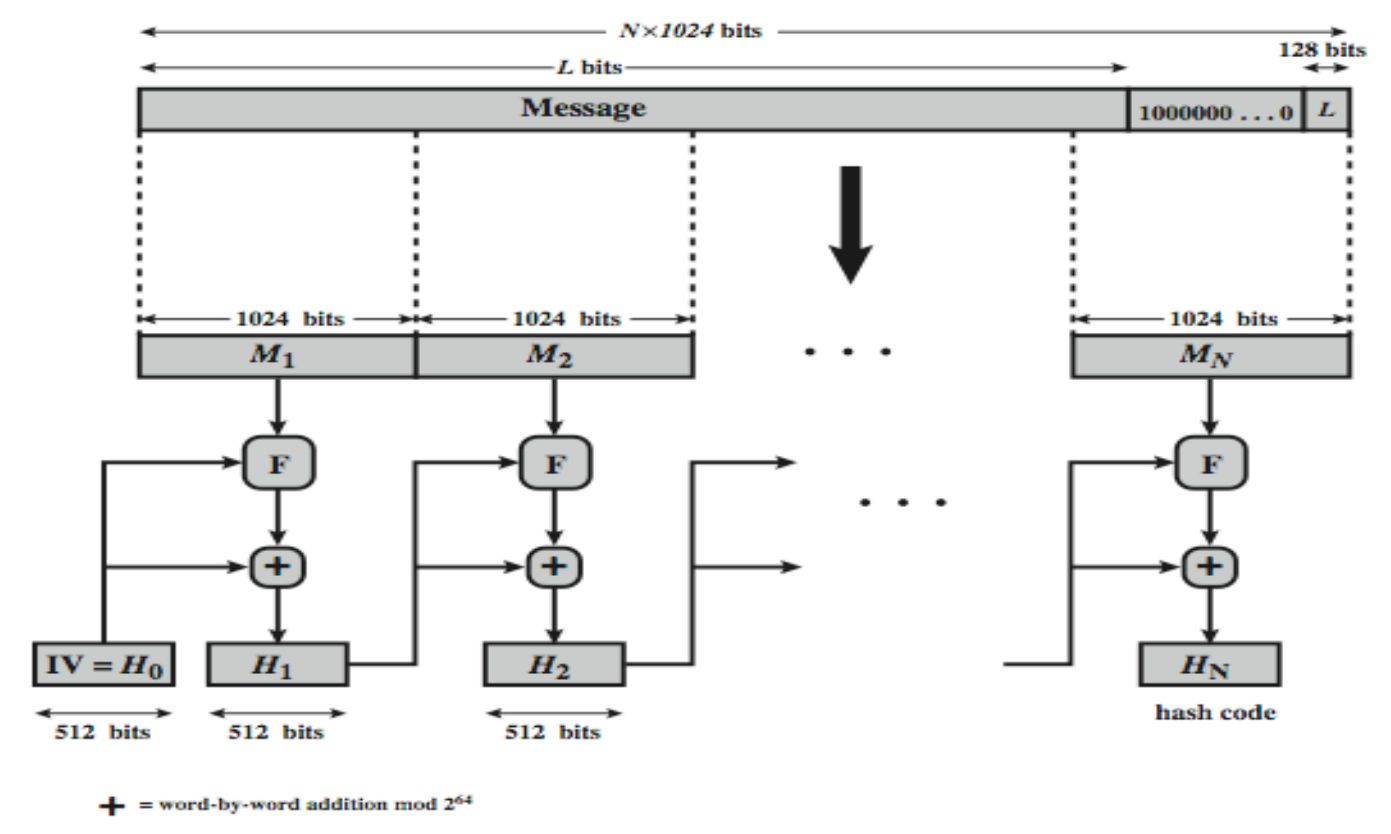
Security

- h is CR, if F is an **ideal cipher**

Well known hash functions

- MD5 (designed in 1991)
  - output 128 bits, collision resistance **completely broken** by researchers in 2004
  - today (controlled) collisions can be found in less than a minute on a desktop PC
- SHA1 – the Secure Hash Algorithm (series of algorithms standardized by NIST)
  - output 160 bits, considered **insecure** for collision resistance
  - **broken** in 2017 by researchers at CWI
- SHA2 (SHA-224, SHA-256, SHA-384, SHA-512)
  - outputs 224, 256, 384, and 512 bits, respectively, **no real security concerns yet**
  - based on Merkle-Damgård + Davies-Meyer generic transforms
- SHA3 (Kessac)
  - **completely new philosophy** (sponge construction + unkeyed permutations)

SHA-2-512 overview



Current hash standards

Algorithm	Maximum Message Size (bits)	Block Size (bits)	Rounds	Message Digest Size (bits)
MD5	2^{64}	512	64	128
SHA-1	2^{64}	512	80	160
SHA-2-224	2^{64}	512	64	224
SHA-2-256	2^{64}	512	64	256
SHA-2-384	2^{128}	1024	80	384
SHA-2-512	2^{128}	1024	80	512
SHA-3-256	unlimited	1088	24	256
SHA-3-512	unlimited	576	24	512

6.3 Generic attacks

Generic attacks against cryptographic hashing

Assume a CR function $h : \{0,1\}^* \rightarrow \{0,1\}^n$

- **brute-force** attack
  - for x = 0 to 2^n-1 (sequentially, for each string x in the domain):
    - compute & record hash value h(x)
    - if h(x) equals a previously recorded hash h(y) halt & output collision on $x \neq y$
- **birthday** attack
  - surprisingly, a more efficient generic attack exists!

Birthday paradox

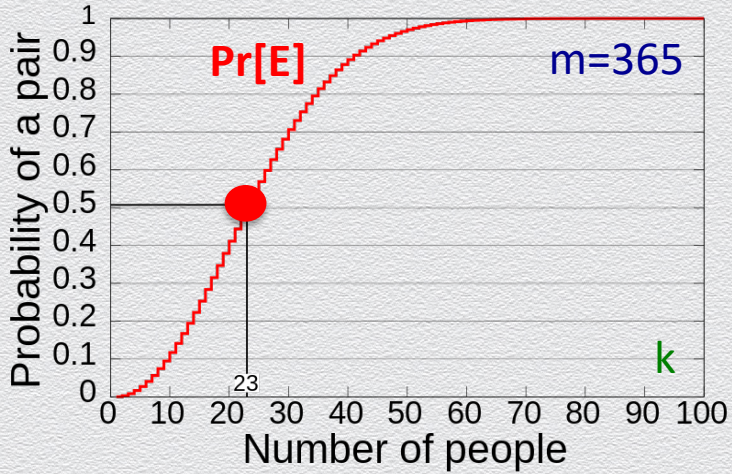
"In any group of 23 people (or more), it is **more likely** (than not) that **at least two** individuals have their birthday on the **same** day"

- based on probabilistic analysis of a random "balls-into-bins" experiment:

  "k balls are each, independently and randomly, thrown into one out of m bins"

- captures likelihood that event E = "**two balls land into the same bin**" occurs

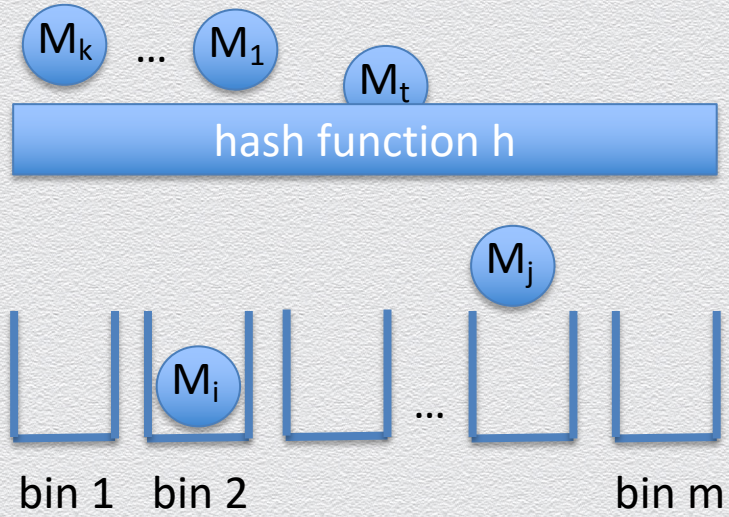
- analysis shows: $\Pr[E] \approx 1 - e^{-k(k-1)/2m}$ (1)
  - if **Pr[E] = 1/2**, Eq. (1) gives **$k \approx 1.17\, m^{1/2}$**
  - thus, for m = 365, k is around 23 (!)
    - assuming a uniform birth distribution



Birthday attack

Applies “birthday paradox” against cryptographic hashing

- exploits the likelihood of finding collisions for hash function h using a **randomized** search, rather than an **exhausting** search
- analogy
  - k balls: distinct messages chosen to hash
  - m bins: number of possible hash values
  - independent & random throwing
    - random message selection + hash mapping



Probabilistic analysis

Experiment

- k balls are each, independently and randomly, thrown into one out of m bins

Analysis

- the probability that the i-th ball lands in an empty bin is: **$1 - (i - 1)/m$**
- the probability F_k that after k throws, no balls land in the same bin is:

$$F_k = (1 - 1/m)\,(1 - 2/m)\,(1 - 3/m)\,\ldots\,(1 - (k - 1)/m)$$

- by the standard approximation $1 - x \approx e^{-x}$: $F_k \approx e^{-(1/m + 2/m + 3/m + \ldots + (k-1)/m)} = e^{-k(k-1)/2m}$
- thus, two balls land in same bin with probability $Pr[E] = 1 - F_k = 1 - e^{-k(k-1)/2m}$
- **lower bound** – Pr[E] increases if the bin-selection distribution is not uniform

What birthday attacks mean in practice…

- # hash evaluations for finding collisions on n-bit digests with probability p

Bits n	Possible outputs (2 s.f.) (H) m	Desired probability of random collision (2 s.f.) (p)									
		10^{-18}	10^{-15}	10^{-12}	10^{-9}	10^{-6}	0.1%	1%	25%	50%	75%
16	65,536	<2	<2	<2	<2	<2	11	36	190	300	430
32	4.3×10^{9}	<2	<2	<2	3	93	2900	9300	50,000	77,000	110,000
64	1.8×10^{19}	6	190	6100	190,000	6,100,000	1.9×10^{8}	6.1×10^{8}	3.3×10^{9}	5.1×10^{9}	7.2×10^{9}
128	3.4×10^{38}	2.6×10^{10}	8.2×10^{11}	2.6×10^{13}	8.2×10^{14}	2.6×10^{16}	8.3×10^{17}	2.6×10^{18}	1.4×10^{19}	2.2×10^{19}	3.1×10^{19}
256	1.2×10^{77}	4.8×10^{29}	1.5×10^{31}	4.8×10^{32}	1.5×10^{34}	4.8×10^{35}	1.5×10^{37}	4.8×10^{37}	2.6×10^{38}	4.0×10^{38}	5.7×10^{38}
384	3.9×10^{115}	8.9×10^{48}	2.8×10^{50}	8.9×10^{51}	2.8×10^{53}	8.9×10^{54}	2.8×10^{56}	8.9×10^{56}	4.8×10^{57}	7.4×10^{57}	1.0×10^{58}
512	1.3×10^{154}	1.6×10^{68}	5.2×10^{69}	1.6×10^{71}	5.2×10^{72}	1.6×10^{74}	5.2×10^{75}	1.6×10^{76}	8.8×10^{76}	1.4×10^{77}	1.9×10^{77}

- for $m = 2^n$, average # hash evaluations before finding the first collision is

$$1.25(m)^{1/2} = 1.25 \times 2^{n/2}$$

Overall

Assume a CR function h producing hash values of size n

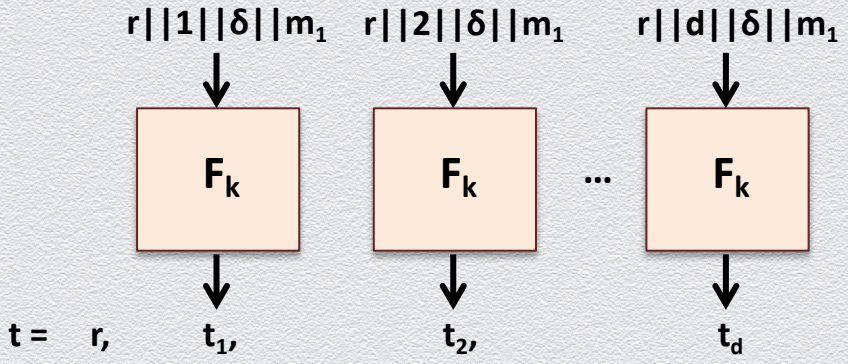
- **brute-force** attack
  - evaluate h on $2^n + 1$ distinct inputs, enumerated by counting
  - by the "pigeon hole" **principle**, at least 1 collision **will be** found
- **birthday** attack
  - evaluate h on (much) **fewer** distinct **randomly** selected inputs
  - by "balls-into-bins" **probabilistic analysis**, at least 1 collision will **more likely** be found
  - when hashing **only** $2^{n/2}$ distinct random inputs, it's **more likely** to find a collision!
  - thus, achieve **N-bit security**, we need **hash values of length** (**at least**) **2N**

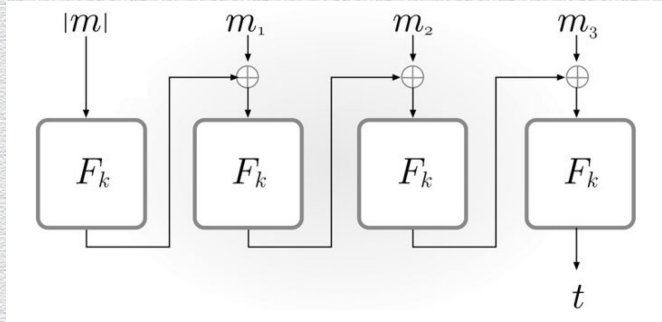
6.4 Applications to cryptography

Hash functions enable efficient MAC design!

Back to problem of designing **secure MAC for messages of arbitrary lengths**

- so far, we have seen two solutions
  - block-based "tagging"
    - based on PRFs
    - inefficient
  - CBC-MAC
    - also based on PRFs
    - more efficient

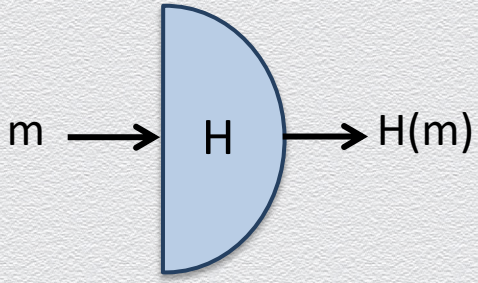


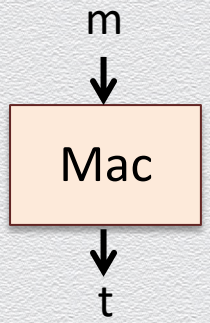


[1] Hash-and-MAC: Design

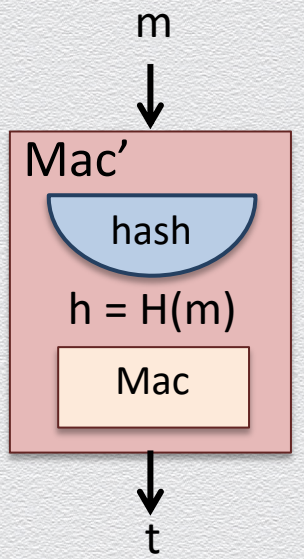
Generic method for designing **secure MAC for messages of arbitrary lengths**

- based on **CR hashing** and **any fix-length secure MAC**





- new MAC (Gen', Mac', Vrf') **as the name suggests**
  - Gen': **instantiate** H and Mac_k with key k
  - Mac': **hash** message m into h = H(m), output **Mac_k**-tag t on h
  - Vrf': **canonical** verification



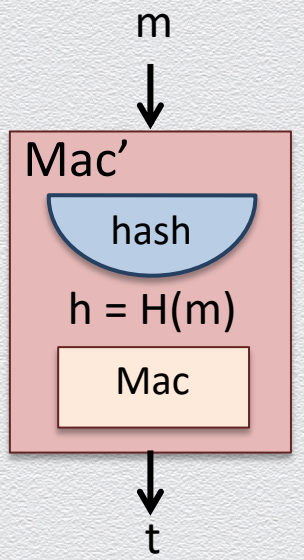
[1] Hash-and-MAC: Security

The Hash-and-MAC construction is a secure as long as

- H is **collision resistant**; and
- the underlying MAC is **secure**

Intuition

- since **H is CR**:
  authenticating **digest H(m)** is **a good as** authenticating **m itself**!



[2] Hash-based MAC

- so far, MACs are based on block ciphers
- can we construct a MAC based on CR hashing?

[2] A naïve, insecure, approach

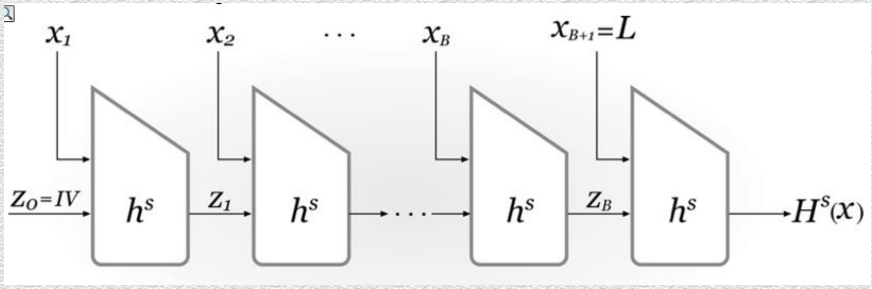
Set tag t as:

$$Mac_k(m) = \mathbf{H}(k||\mathbf{m})$$

- intuition: given H(k||m) it should be infeasible to compute H(k||m'), m' ≠ m

Insecure construction

- practical CR hash functions employ the Merkle-Damgård design
- **length-extension attack**
  - knowledge of $H(m_1)$ makes it feasible to compute $H(m_1||m_2)$
  - by knowing the length of m_1, one can learn internal state z_B even without knowing m_1!

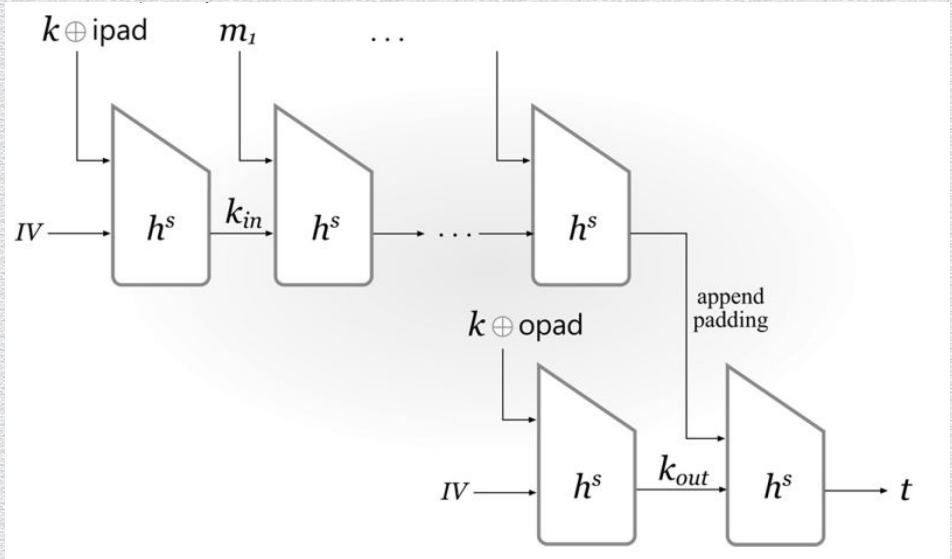


[2] HMAC: Secure design

Set tag t as:

$$HMAC_k[m] = H\,[\ (k \oplus opad)\ ||\ H[\ (k \oplus ipad)\ ||\ m\ ]\ ]$$

- intuition: **instantiation of hash & sign paradigm**
- two layers of hashing H
  - **upper layer**
    - y = H((k ⊕ ipad) || m)
    - y = H'(m), i.e., "hash"
  - **lower layer**
    - t = H ((k ⊕ opad) || y')
    - t = Mac'(k_{out}, y'), i.e., "sign"



[2] HMAC: Security

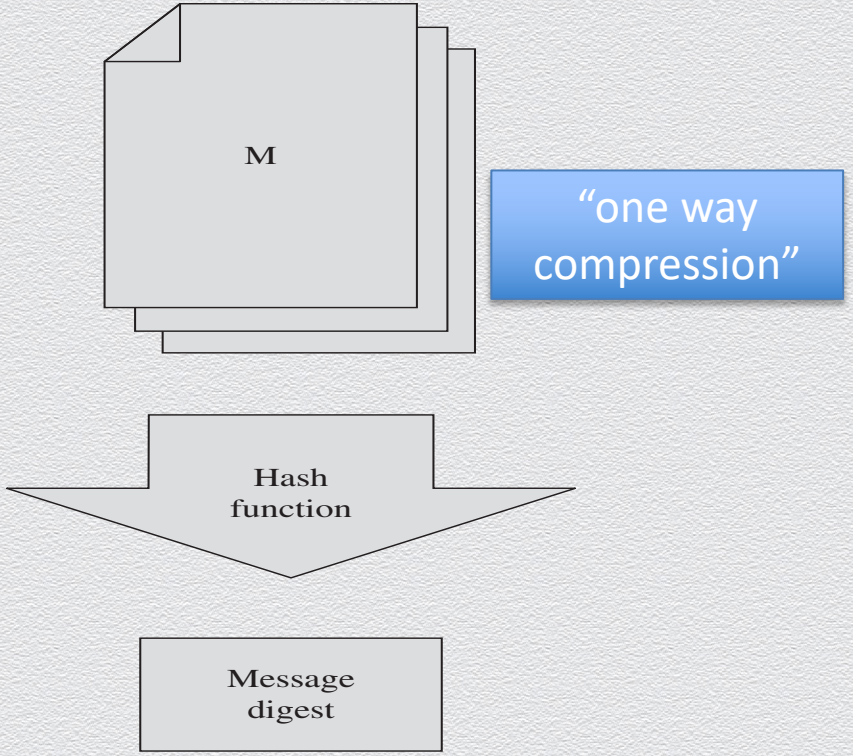
If used with a secure hash function and according to specs, HMAC is secure

- no practical attacks are known against HMAC

6.5 Applications to security

Generally: Message digests

Short secure description of data primarily used to detect changes



Application 1: Digital envelops

Commitment schemes

- two operations
- commit(x, r) = C
  - i.e., put message x into an envelop (using randomness r)
  - commit(x, r) = h(x || r)
  - **hiding property**: you cannot see through an (opaque) envelop
- open(C, m, r) = ACCEPT or REJECT
  - i.e., open envelop (using r) to check that it has not been tampered with
  - open(C, m, r): check if h(m || r) =? C
  - **binding property**: you cannot change the contents of a sealed envelop

Application 1: Security properties

Hiding: perfect opaqueness

- similar to indistinguishability; commitment reveals nothing about message
  - adversary selects two messages x_1, x_2 which he gives to challenger
  - challenger randomly selects bit b, computes (randomness and) commitment C_i of x_i
  - challenger gives C_b to adversary, who wins if he can find bit b (better than guessing)

Binding: perfect sealing

- similar to unforgeability; cannot find a commitment "collision"
  - adversary selects two distinct messages x_1, x_2 and two corresponding values r_1, r_2
  - adversary wins if commit(x_1, r_1) = commit(x_2, r_2)

Example 1: Fair digital coin flipping

Problem

- To decide who will do the dishes: Alice is to call the coin flip & Bob is to flip the coin
- But Alice may change her mind, Bob may skew the result

Protocol

- 1. Alice calls the coin flip but only tells Bob a commitment to her call
- 2. Bob flips the coin & reports the result
- 3. Alice reveals what she committed to & Bob verifies that Alice's call matches her commitment
- If Alice’s revealed commitment matches Bob’s reported result, Alice wins; else Bob wins

Example 1: Fair digital coin flipping (cont.)

Protocol

- 1. Alice calls the coin flip but only tells Bob a commitment to her call
- 2. Bob flips the coin & reports the result
- 3. Alice reveals what she committed to & Bob verifies that Alice's call matches her commitment
- If Alice’s revealed commitment matches Bob’s reported result, Alice wins; else Bob wins

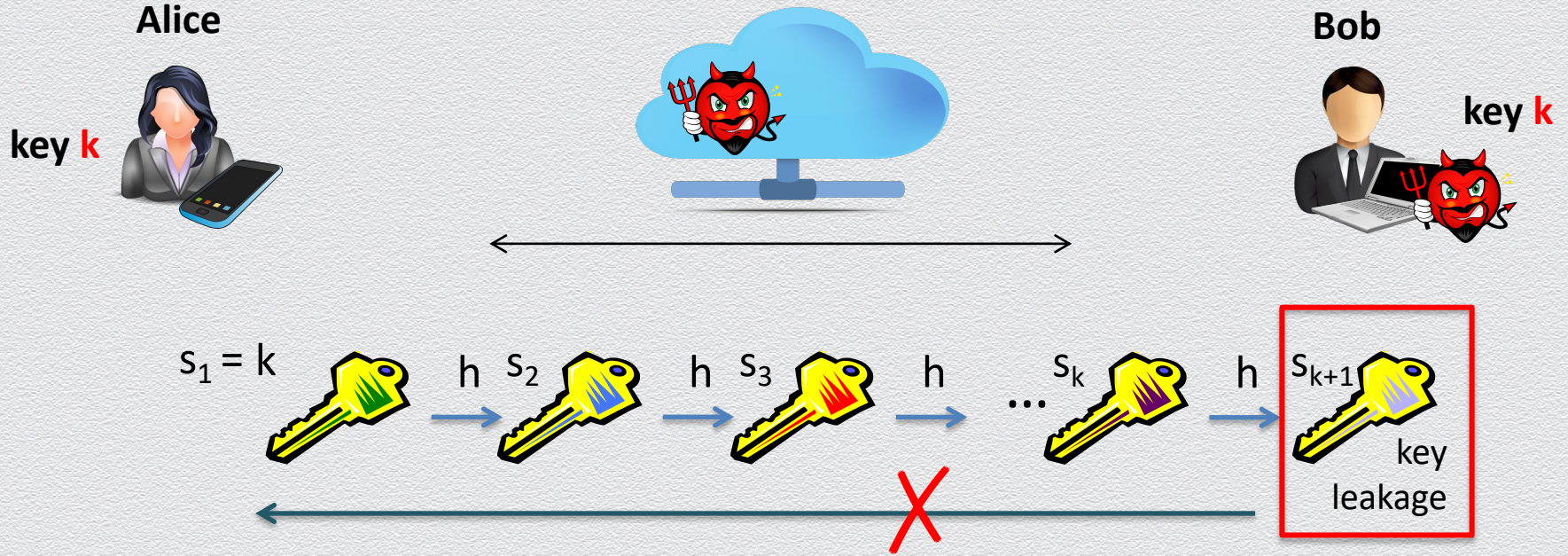
Security

- Hiding: Bob does not get any advantage by seeing Alice’s commitment
- Binding: Alice cannot change her mind after the coin is flipped

Application 2: Forward-secure key rotation

Alice and Bob secretly communicate using symmetric encryption

- Eve intercepts their messages and later breaks into Bob's machine to steal the shared key



Application 3: Hash values as file identifiers

Consider a cryptographic hash function H applied on a file F

- the hash (or digest) H(M) of F serves as a **unique** identifier for F
  - "uniqueness"
    - if another file F' has the same identifier, this contradicts the security of H
  - thus
    - the hash H(F) of F is like a fingerprint
    - one can check whether two files are equal by comparing their digests

Many real-life applications employ this simple idea!

Examples

3.1 Virus fingerprinting

- When you perform a virus scan over your computer, the virus scanner application tries to identify and block or quarantine programs or files that contain viruses
- This search is primarily based on comparing the digest of your files against a database of the digests of already known viruses
- The same technique is used for confirming that is safe to download an application or open an email attachment

3.2 Peer-to-peer file sharing

- In distributed file-sharing applications (e.g., systems allowing users to contribute contents that are shared amongst each other), both shared files and participating peer nodes (e.g., their IP addresses) are uniquely mapped into identifiers in a hash range
- When a given file is added in the system it is consistently stored at peer nodes that are responsible to store files those digests fall in a certain sub-range
- When a user looks up a file, routing tables (storing values in the hash range) are used to eventually locate one of the machines storing the searched file

Example 3.3: Data deduplication

Goal: Elimination of duplicate data

- Consider a cloud provider, e.g., Gmail or Dropbox, storing data from numerous users.
- A vast majority of stored data are duplicates; e.g., think of how many users store the same email attachments, or a popular video…
- Huge cost savings result from deduplication:
  - a provider stores identical contents possessed by different users once!
  - this is completely transparent to end users!

Idea: Check redundancy via hashing

- Files can be reliably checked whether they are duplicates by comparing their digests.
- When a user is ready to upload a new file to the cloud, the file's digest is first uploaded.
- The provider checks to find a possible duplicate, in which case a pointer to this file is added.
- Otherwise, the file is being uploaded literally
- This approach saves both storage and bandwidth!

Application 4: Concealing stored passwords

Goal: User authentication

- Today, passwords are the dominant means for user authentication, i.e., the process of verifying the identity of a user (requesting access to some computing resource).
- This is a “something you know” type of user authentication, assuming that only the legitimate user knows the correct password.
- When you provide your password to a computer system (e.g., to a server through a web interface), the system checks if your submitted password matches the password that was initially stored in the system at setup.

Problem: How to protect password files

- If password are stored at the server in the clear, an attacker can steal the password file after breaking into the authentication server – this type of attack happens routinely nowadays…
- Password hashing involved having the server storing the hashes of the users passwords.
- Thus, even if a password file leaks to an attacker, the onewayness of the used hash function can guarantee some protections against user-impersonation simply by providing the stolen password for a victim user.

Example 4: Password storage

Identity	Password
Jane	qwerty
Pat	aaaaaa
Phillip	oct31witch
Roz	aaaaaa
Herman	guessme
Claire	aq3wm$oto!4

Plaintext

Identity	Password
Jane	0x471aa2d2
Pat	0x13b9c32f
Phillip	0x01c142be
Roz	0x13b9c32f
Herman	0x5202aae2
Claire	0x488b8c27

Concealed via hashing

Application 5: Hash-and-digitally-sign

Very often digital signatures are used with hash functions

- the hash of a message is signed, instead of the message itself

Signing message M

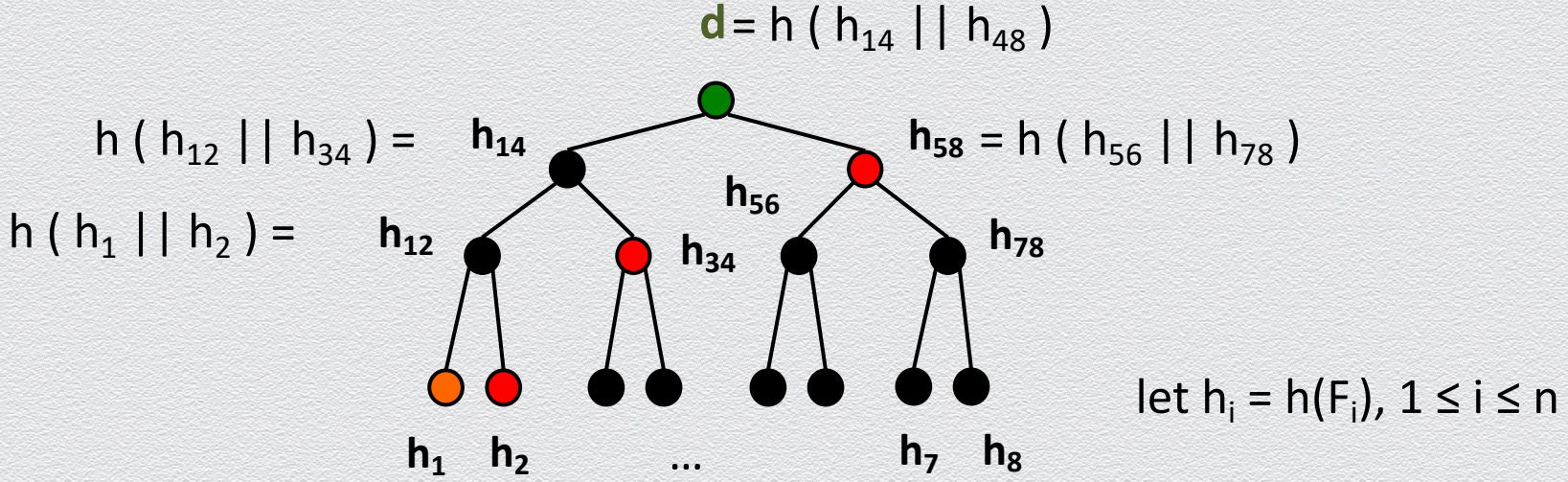
- let h be a cryptographic hash function, assume RSA setting (n, d, e)
- compute signature $\sigma = h(M)^d \bmod n$
- send σ, M

Verifying signature σ

- use public key (e,n)
- compute $H = \sigma^e \bmod n$
- if H = h(M) output ACCEPT, else output REJECT

Application 6: The Merkle tree

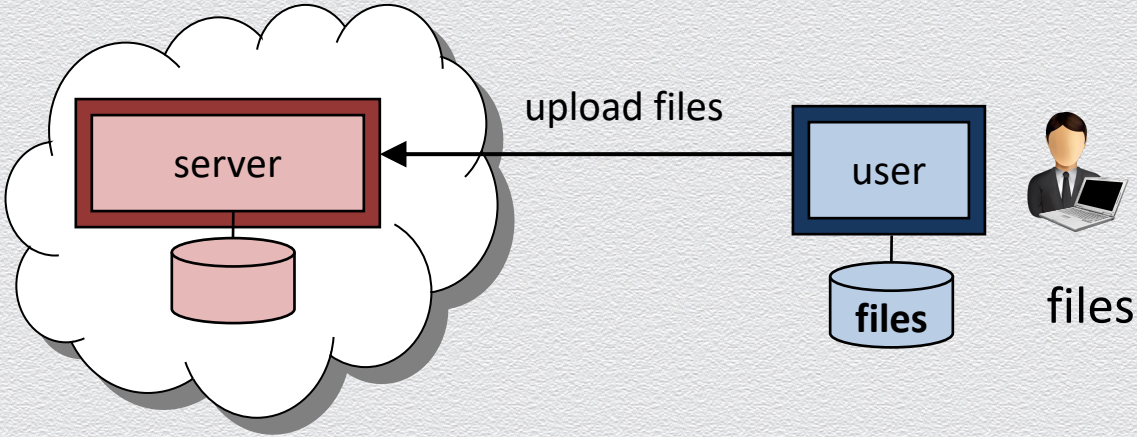
- an alternative (to Merkle-Damgård) method to achieve domain extension



Motivation: Secure cloud storage

- Bob has files $f_1, f_2, \ldots, f_n$
- Bob sends to Amazon S3 (cloud storage service)
  - the hashes $h(r||f_1), h(r||f_2), \ldots, h(r||f_n)$
  - files $f_1, f_2, \ldots, f_n$
- Bob stores randomness r (and keeps it secret)
- Every time Bob **reads** a file f_1, he also reads $h(r||f_i)$ and verifies f_1 integrity
- Any problems with **writes**?

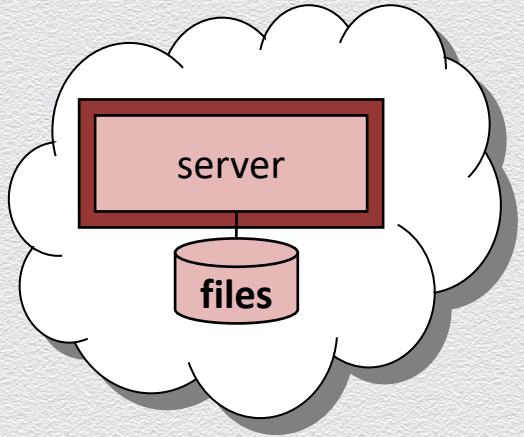
Cloud storage model

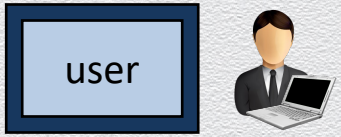


(1)

files = (F1, F2, …, F7, F8)

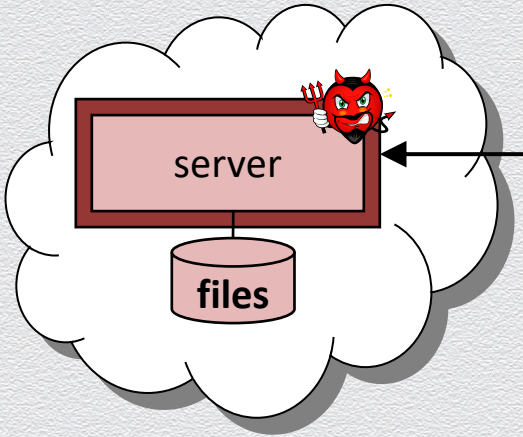
(2)





Cloud storage model

- attack by malicious server

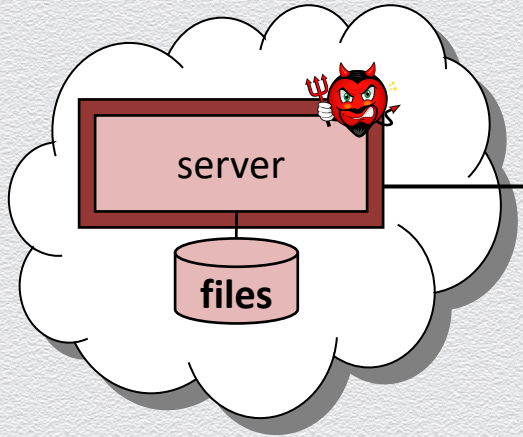


give me
file F1

(3)

files = (F1, F2, …, F7, F8)

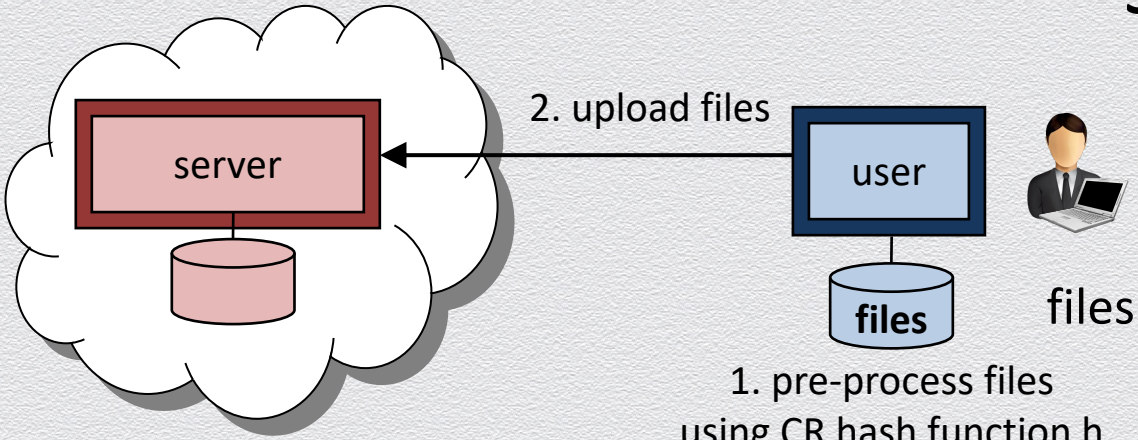
(4)



here it is, F1’

F1’ = #@$@!#^@$^… (altered)

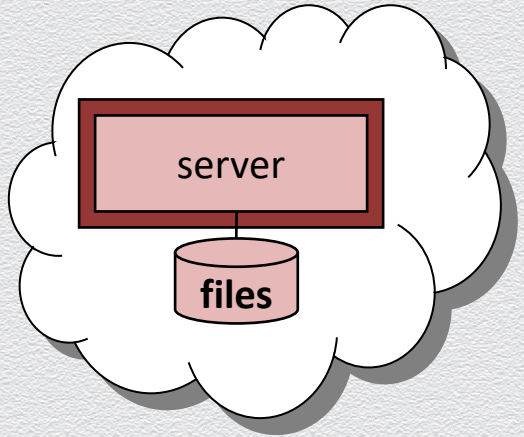
Secure cloud storage model
- integrity protection via hashing

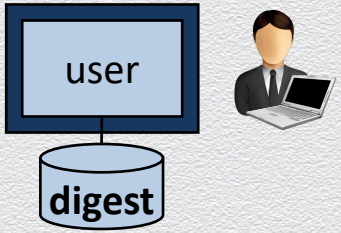


(5)

files = F = (F1, F2, …, F7, F8)

(6)

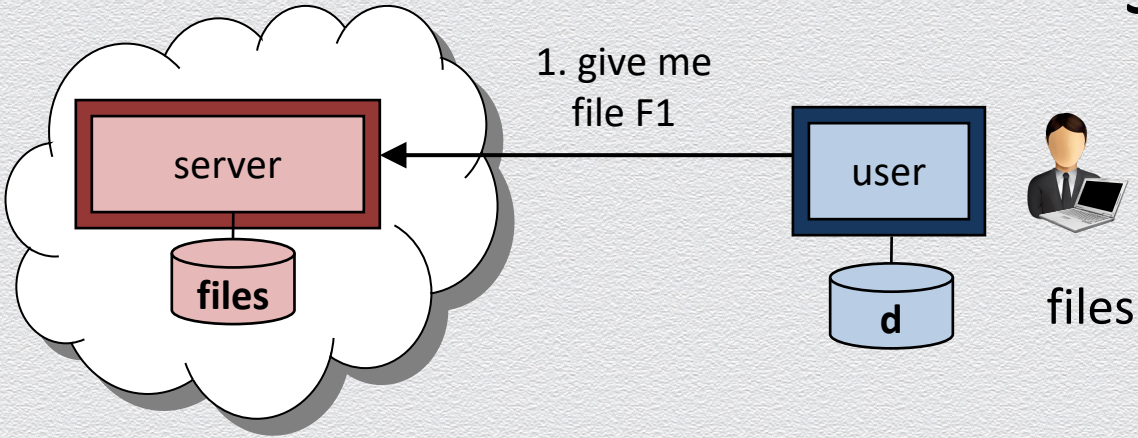




digest d is computed over all files

|d| << |F|

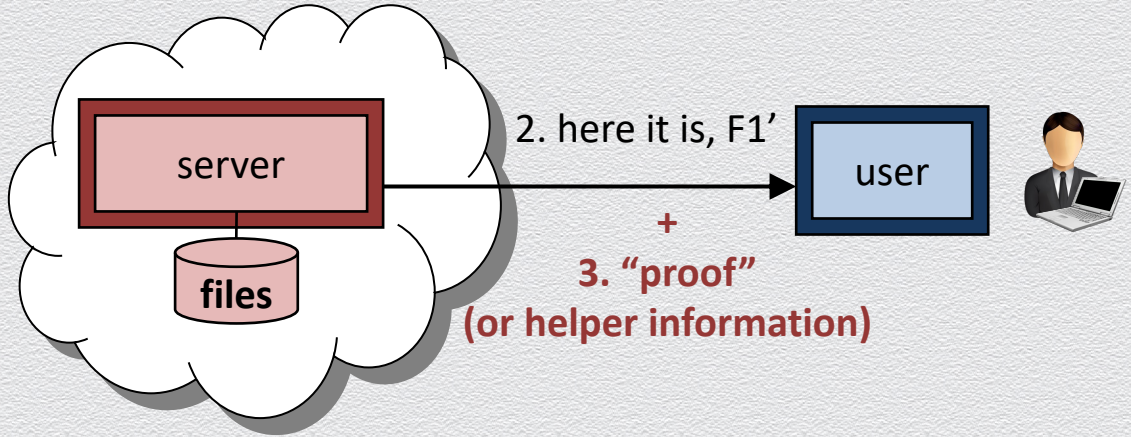
Secure cloud storage model
- how verification works



(7)

files = (F1, F2, …, F7, F8)

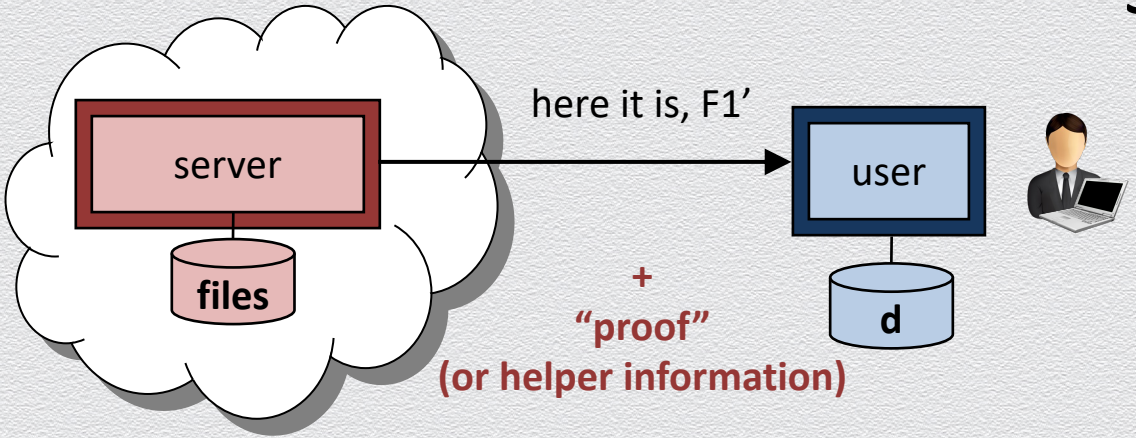
(8)



4. verification

"is F1' intact?"

Secure cloud storage model
- verification via hashing



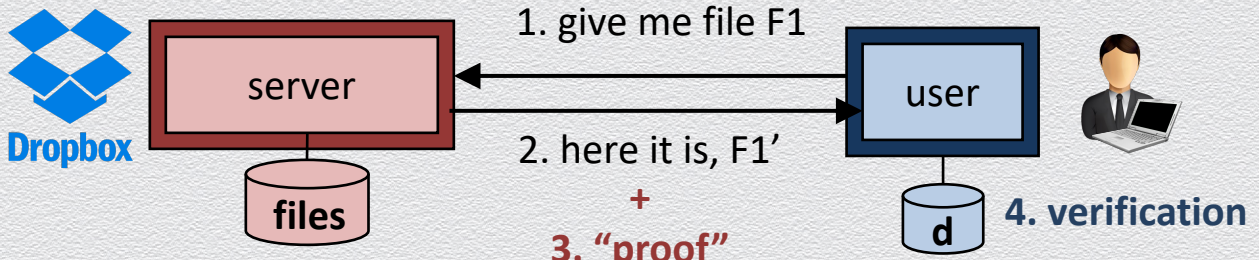
(9)

verification

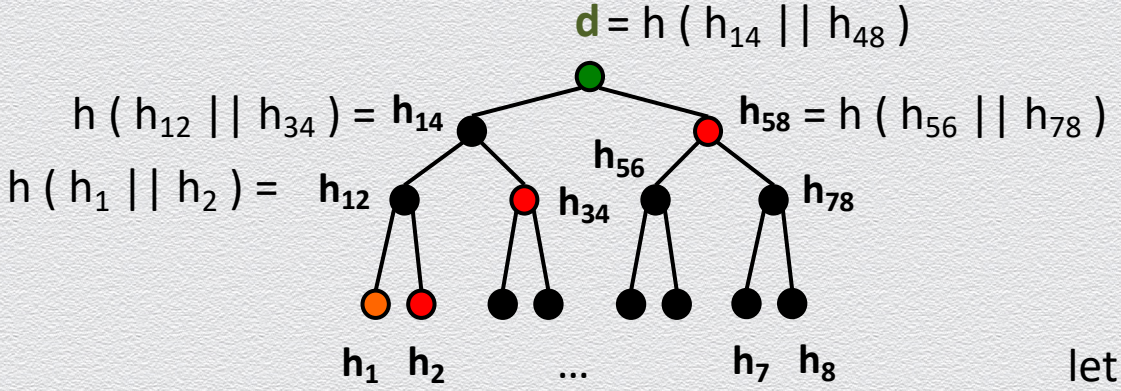
"is F1' intact?"

- user has
  - authentic digest d (locally stored)
  - file F1' (to be checked/verified as it can be altered)
  - **proof** (to help checking integrity, but it can be maliciously chosen)
- verification involves (performed locally at user)
  - combine the file F1' with the proof to re-compute candidate digest d'
  - check if d' = d
  - if yes, then F1 is intact; otherwise tampering is detected!

Overall: Data authentication via the Merkle tree



$F = (F_1, F_2, \ldots, F_8)$



let $h_i = h(F_i)$, $1 \le i \le n$